Flexure of the Lithosphere

- **What is the basic form of flexure in the lithosphere ?**
- **Some natural examples**
- **What are the governing equations for flexure ?**
- **Work some examples ...**

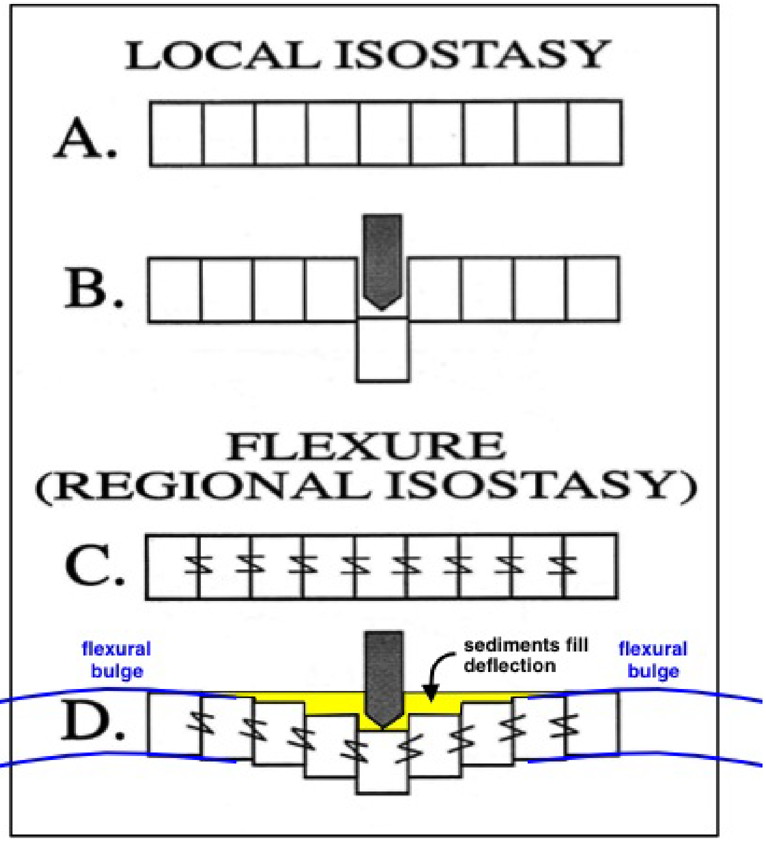


Figure 2.4 Conceptual comparison of local isostasy versus flexure (regional isostasy).

A. In local isostasy, the lithosphere is composed of separate blocks. As a load is placed on the surface of the earth **(B)**, only the block immediately beneath the load subsides.

C. The earth (often) has lateral strength, as if the blocks are attached to each other by springs. Emplacement of a load on the surface of the earth causes subsidence **(D)**, which is compensated over a larger area due to the rigidity of the lithosphere.

Angevine et al. (1990)

Flexural Subsidence Defined:

Distributed deflection, or bending, of the lithosphere in response to an applied vertical load. Subsidence is distributed over an area wider than the load itself because the lithosphere has flexural rigidity (lateral strength).

For the case of a two-dimensional load distribution, such as a linear mountain belt or rift system, the elastic flexure equation reduces to:

intraplate stress

ρ_a *or* $(\rho_a-\rho_s)$

bending moment included here

$$D\frac{d^4 w}{d x^4} + N\frac{d^2 w}{d x^2} + \rho_m g w = \boxed{p(x)} \qquad (5.2)$$

vertical load

where x = distance (normal to load axis) w = vertical deflection of the crust
D = flexural rigidity
N = intraplate force (positive if compressive) = *horizontal load*
p = vertical load distribution

Angevine et al. (1990)

Key Assumptions when using this general equation:

1. **The lithosphere has a "linear" – simple – *elastic* rheology** (define this term)
2. **Two-dimensional load distribution** (no significant changes in or out of x-section)
3. **Elastic lithosphere is thin relative to horizontal dimensions**
4. **Vertical deflections are small relative to horizontal dimensions**
5. **Flexural rigidity (D) is constant across the x-section** (this can be relaxed, then eqn gets complicated)

Examples in Nature – 1: Loading at subduction zones

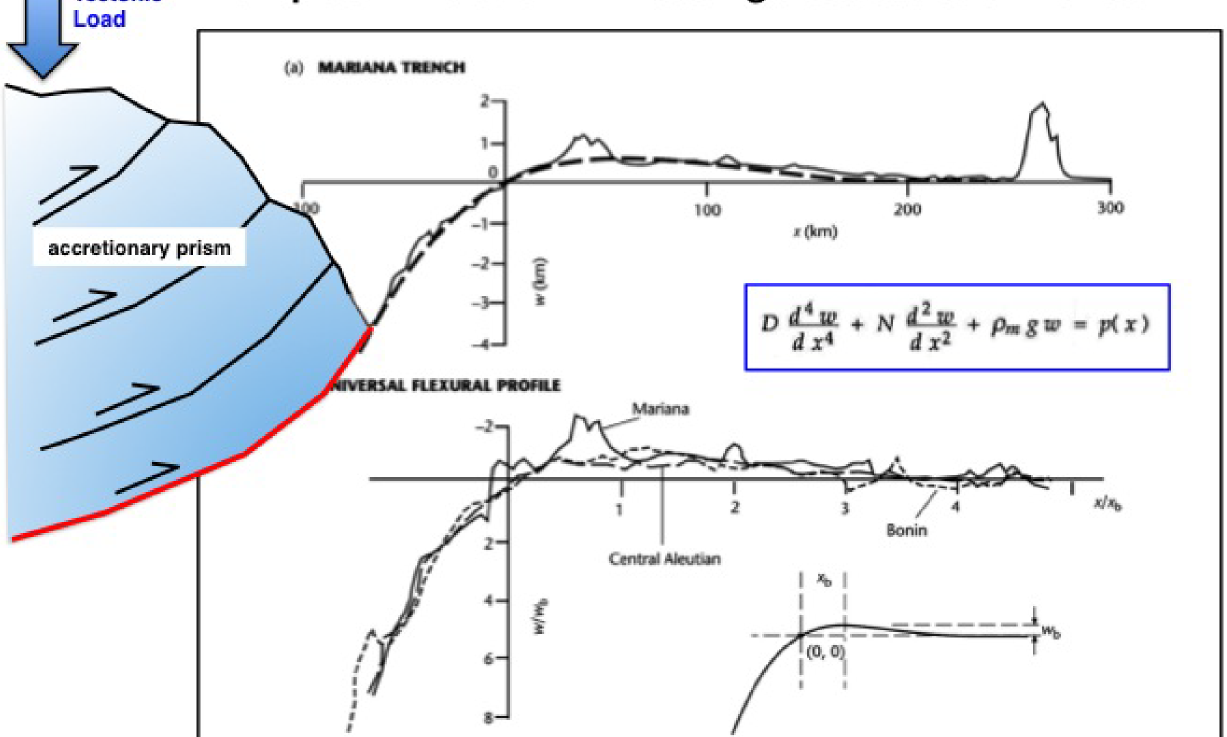


Fig. 4.5 (a) Bathymetry of the Mariana Trench (Watts & Talwani 1974) compared with that predicted by the universal flexural equation with x_b taken as 55 km and w_b as 0.5 km (after Turcotte & Schubert 1982, p. 130; © Cambridge University Press, 2002). (b) Non-dimensional comparison of the bathymetric profile of the Mariana, Bonin and Central Aleutian oceanic trenches with a universal flexural profile (after Caldwell *et al.* 1976; reprinted with permission of Elsevier). Deflection w is scaled by the maximum elevation of the bulge w_b, and the horizontal coordinate x is scaled by the horizontal distance to the crest of the bulge x_b. The x-coordinate is zeroed at the first node of the deflection x_0 where $w = 0$ (inset).

Allen & Allen (2013)

Examples in Nature – 2: Loading at volcanic islands

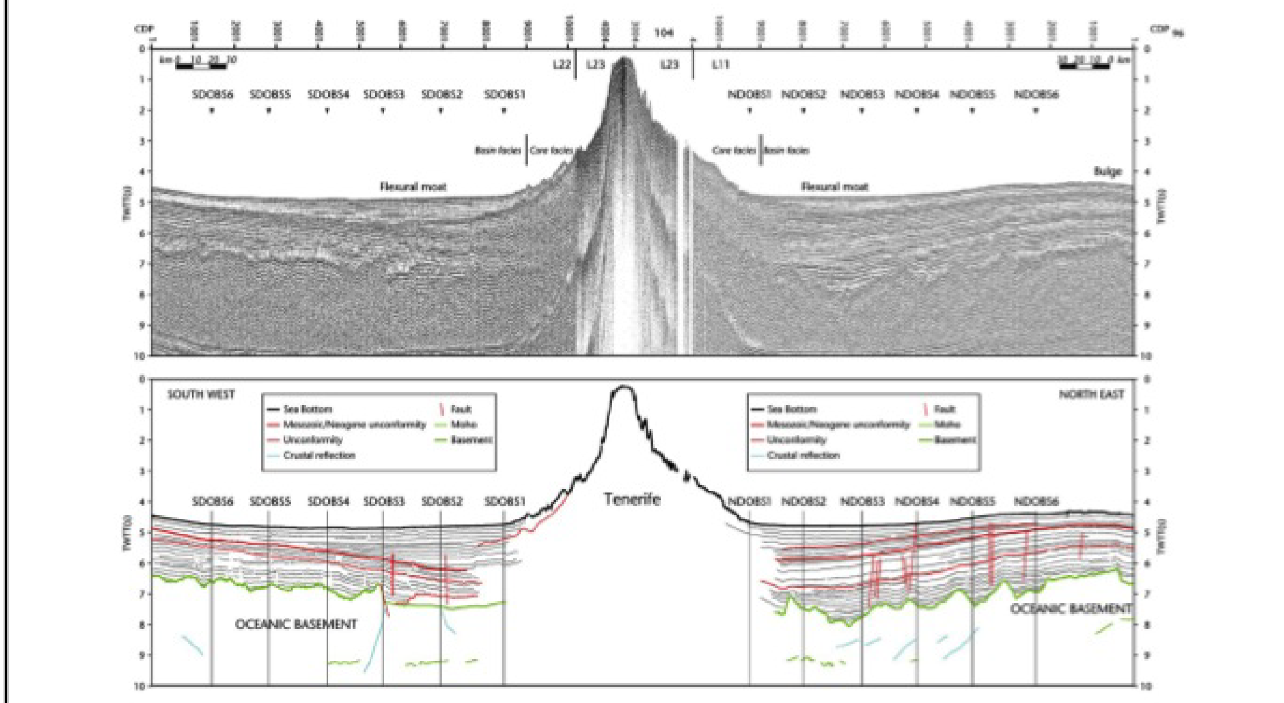


Fig. 4.4 Cross-section of Tenerife, Canary Islands, based on seismic reflection data, showing deflection of the ocean crust and the structure of the sedimentary infill of the moat on each side of the island line load. Youngest stratigraphy shows thickening towards load, indicating active volcanic island growth and flexure of the oceanic plate. Reprinted from Watts *et al.* (1997), with permission of Elsevier.

Allen & Allen (2013)

3. Apennine foredeep basin

Compare Flexural Profiles

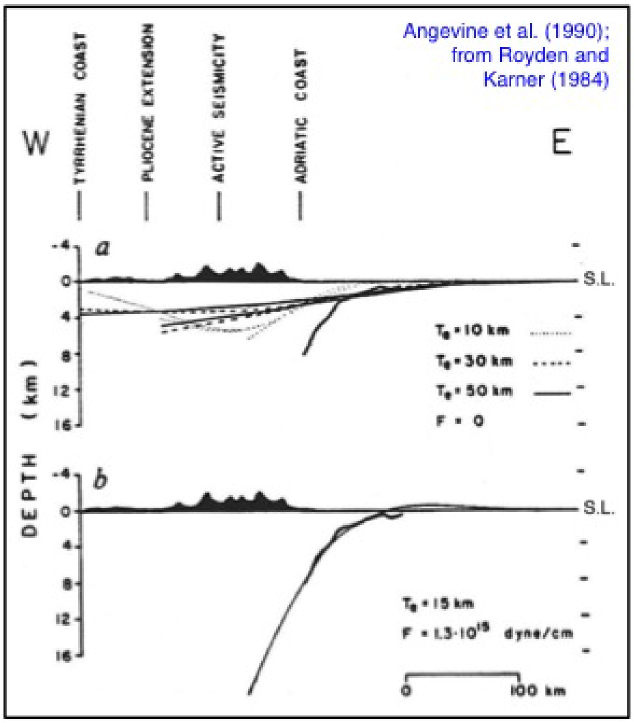


Figure 5.12 Deflection of a basal Pliocene horizon (heavy curve) in the Apennine foredeep basin (from Royden and Karner, 1984).
(A) The topographic load of the thrust belt is too small to explain the observed deflection. (B) A line load is applied at the end of a hypothesized broken plate, under the Apennines, to make the calculated (model) deflection match the observed deflection.

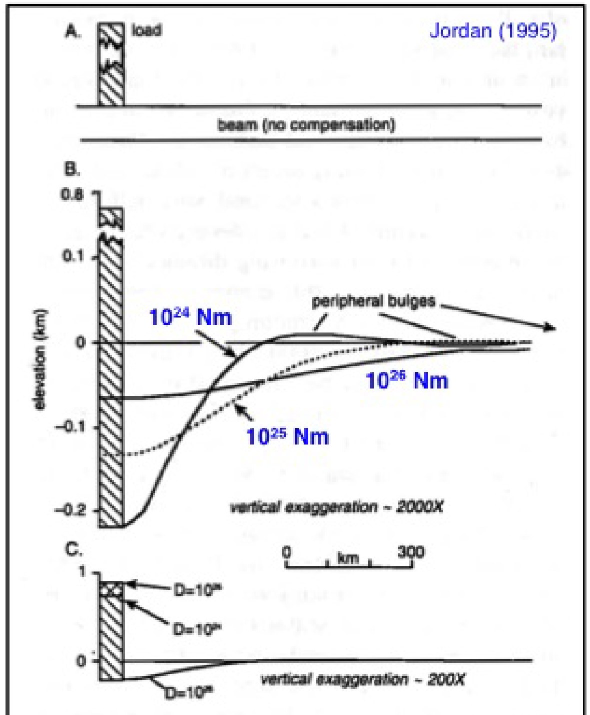


Fig. 9.4 Cross section of the deformation of an elastic beam (after Beaumont, 1981). A) Configuration of load and beam, in situation with no isostatic compensation. B) Regional compensation of a 50 km-wide by 1 km-thick load, showing that the deflected profile of the basin depends on the flexural rigidity, D, of the plate (units of D are Newton-meters). C) Same deformation as in B, but with less vertical exaggera-

General Equations to calculate vertical deflection of an elastic plate.

Below are some parameters commonly used in solving this problem.

(from Turcotte and Schubert, 1982)

Effective elastic thickness (EET):

$$\mathrm{EET} = \sqrt[3]{\frac{12(1-\nu^2)D}{E}}$$

Equivalent thickness of elastic plate

ν = Poisson's ratio (0.25)

E = Young's Modulus (7×10^{10} N/m^2)

Flexural Parameter (α): α in units of distance (km) ... prop. to D (flex. rigidity)

$$\alpha = \sqrt[4]{\frac{4D}{\rho_a g}} \quad \text{or} \quad \alpha = \sqrt[4]{\frac{4D}{(\rho_a-\rho_w) g}} \quad \text{or} \quad \alpha = \sqrt[4]{\frac{4D}{(\rho_a-\rho_s) g}}$$

(depending on whether the deflection is filled with air (assume $\rho_{air} = 0$), water (ρ_w) or sediment (ρ_s); $\rho_a = 3300$ kg/m^3; $\rho_s = 2250$ kg/m^3

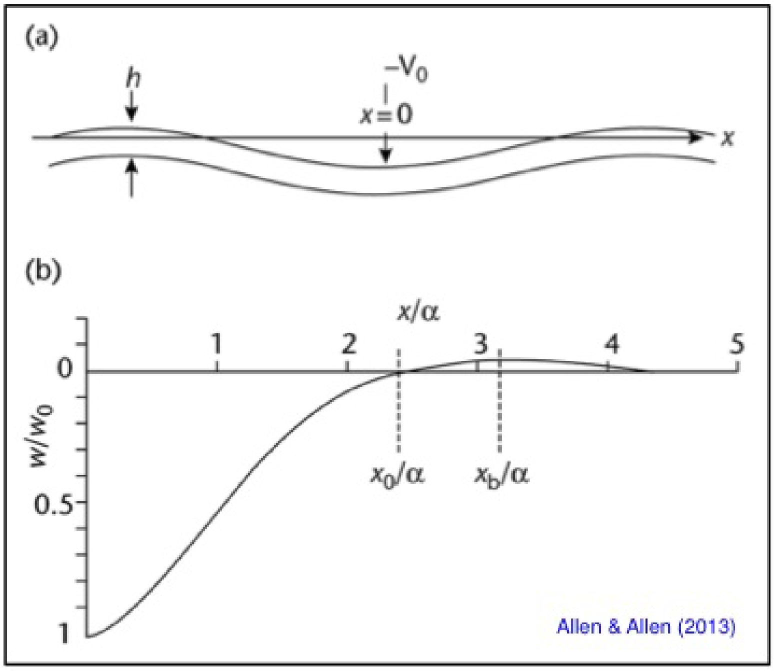
Line Load (V_o): $V_o = \rho_L\, g\, h\, \Delta x$

(see hand-written class notes for derivation)

Solve for vertical deflection of an elastic plate in response to an applied load.

Two popular solutions: (1) for an infinite plate, and (2) broken plate.

(1) Infinite Plate: continuous, unbroken under the load



Infinite Plate

$$w = w_o\, e^{-x/\alpha}\left(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha}\right)$$

$$w_o = \frac{V_o\,\alpha^3}{8\,D}$$ W_o Vertical deflection beneath a line load ($V_o = \rho_L * g * h * \Delta x$)

$$x_o = \frac{3\,\pi}{4}\,\alpha$$ X_o Distance to where deflection (w) = 0

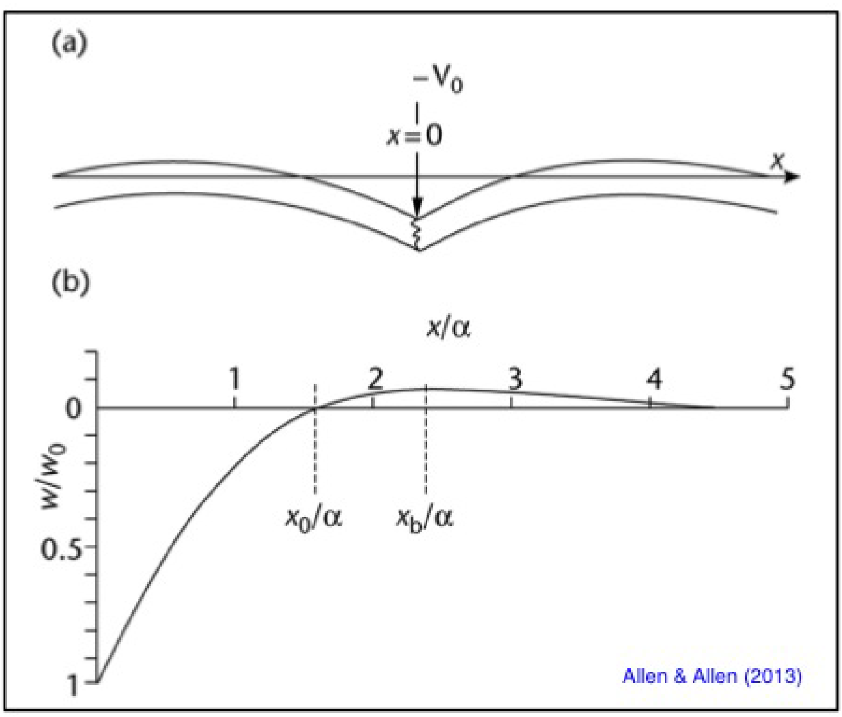
$$x_b = \pi\,\alpha$$ X_b Distance to crest of flexural bulge

$$w_b = -\,w_o\, e^{-\pi}$$ W_b Height of flexural bulge

Turcotte & Schubert (1982)

(2) Boken Plate: Very weak or "broken" under the load

(plate has zero strength at x = 0)



Broken Plate

$$w = w_o\, e^{-x/\alpha}\left(\cos\frac{x}{\alpha}\right)$$

$$w_o = \frac{V_o\,\alpha^3}{4\,D}$$ W_o Vert. deflection at line load

$$x_o = \frac{\pi}{2}\,\alpha$$ X_o Dist. to where deflection (w) = 0

$$x_b = \frac{3\,\pi}{4}\,\alpha$$ X_b Distance to crest of flex. bulge

$$w_b = w_o\, e^{-3\pi/4}\left(\cos\frac{3\pi}{4}\right)$$ (height of flex. bulge)

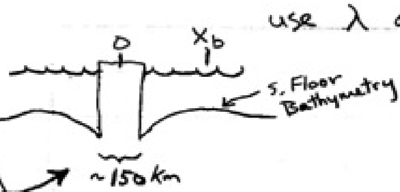
Turcotte & Schubert (1982)

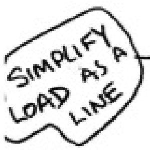
Solve an Example: Hawaiian islands load on ocean lithosphere

Calculate α, D, EET (from simple measurement of X_b)

(1) <u>Assume Infinite Plate</u> (continuous beneath load)

use λ of deflection to calculate α, D, EET





① Assuming <u>INFINITE PLATE</u>

$X_b = 250$ km

$\alpha = \frac{X_b}{\pi} = 79.55 \text{ km} \approx 80{,}000 \text{ m}$

$$D = \frac{\alpha^4 (\rho_m - \rho_w) g}{4} = \frac{(80{,}000 \text{ m})^4 (2300 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}{4}$$

Units: $\left(\frac{\text{Kg m}^2}{\text{S}^2}\right)$

Units: $1 N = \frac{1 \text{ Kg m}}{\text{S}^2} \Rightarrow \frac{\text{Kg m}^2}{\text{S}^2} \times \frac{N}{\text{Kg m/s}^2} = \text{Nm}$

$D = 2.3 \times 10^{23}$ Nm $(= 2.3 \times 10^{30}$ dyn cm$)$

$$EET = \sqrt[3]{\frac{12 D (1-\nu^2)}{E}} = \sqrt[3]{\frac{12 (2.3 \times 10^{23} \text{ Nm})(1 - .25^2)}{7 \times 10^{10} \text{ N/m}^2}} = \boxed{33.3 \text{ Km}}$$

Hawaiian Islands Example

(2) <u>Assume Broken Plate</u> (zero strength beneath load)

again, $X_b = 250$ km

Now, $\alpha = \frac{4 X_b}{3\pi} = 106.1 \text{ km} = 106{,}100 \text{ m}$

$$D = \frac{\alpha^4 (\rho_m - \rho_w) g}{4} = \frac{(106{,}100 \text{ m})^4 (2300 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}{4}$$

$$= 7.14 \times 10^{23} \text{ Nm}$$

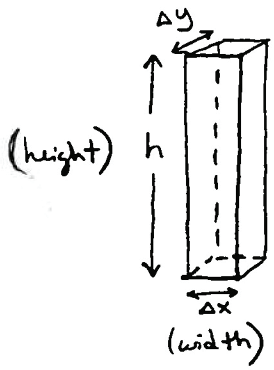
(flex. rigidity)

<u>Note</u>: D calc. for broken plate ~ 3 times D(infin. plate)

<u>Why??</u> Because <u>broken plate</u> is "<u>weaker</u>", No elastic resistance to V_0 is transmitted across $x = 0$. Thus, to produce same width (or λ) of deflection in broken plate, need <u>stronger plate</u> (greater D)

$$EET = \sqrt[3]{\frac{12 D (1-\nu^2)}{E}} = \boxed{48.5 \text{ Km}}$$

Calculating Crustal Loads

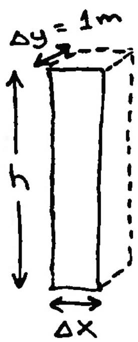


<u>For 3-D Problem:</u>

$$F = (\rho_c)(\text{vol.})(g) \qquad \text{vol.} = (h)(\Delta x)(\Delta y)$$

$$= \left(\frac{kg}{m^3}\right)\left(m^3\right)\left(\frac{m}{s^2}\right)$$

$$= \frac{kg\ m^4}{m^3\ s^2} = \frac{kg\,m}{s^2} = N \quad (\underline{\text{force}})$$



<u>For 2-D problem,</u> set y dimension = to unity (1 m), then evaluate <u>force</u> (N) <u>per</u> <u>unit</u> "<u>thickness</u>" (y, in m), where y is perpendic. to plane of x-section.

$$N/m = \frac{kg\,m/s^2}{m} = \frac{kg}{s^2}$$

So, considering load in 2-D cross-section,

we obtain a vertical load (V):

$$V = (\rho_c)(h)(\Delta x)(g)$$

$$= \left(\frac{kg}{m^3}\right)(m)(m)\left(\frac{m}{s^2}\right)$$

$$= \frac{kg\ m^3}{m^3\ s^2} = \frac{kg}{s^2} = N/m$$

= Force per unit length $\perp$ to plane of cross-section